Proceedings of IAM, V.15, N.1, 2026, pp.3-8
https://doi.org/10.30546/2225-0530.15.01.2026.003

ON THE ABSENCE OF GLOBAL PERIODIC SOLUTIONS TO THE NONLINEAR EVOLUTIONARY SCHRÖDINGER EQUATION

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Abstract. The problem of absence of global periodic solutions to a nonlinear evolutionary Schrödinger equation with a damped linear term is investigated. It is proven that, at the case when the damping coefficient is nonnegative, the problem under study has no global periodic solutions for any initial data, and in the case of a negative damping coefficient, this statement remains valid at sufficiently large values of the initial data.

Keywords: Nonlinear evolution equation, Schrödinger equation, periodic solution, global solution, absence of periodic global solutions.

AMS Subject Classification: 35Q70, 39B72.

1. Introduction

The nonlinear evolutionary Schrödinger equation is being intensively studied in the scientific literature [1,4-6,15]. The study of nonlinear evolutionary equations of the Schrödinger type is also relevant and of scientific interest.

2. Problem Statement

Let $T-$ is the one-dimensional torus . Let consider the following problem:

$$\begin{aligned} &u_t = i\theta u_{xx} + f(u) + \omega u, \; x \in T, t > 0, \\ &u(x,0) = u_0(x), \end{aligned} \tag{1}$$

where $u_0(x)$ is the periodic function with period $L > 0$,

$$u_0(x+L) = u_0(x). \tag{2}$$

Here

$$f(u) = (\eta + i\mu)|u|^{\rho+1}, \tag{3}$$

where $\eta, \mu, \omega -$ are real numbers, $\theta \neq 0$, $\rho > 0, \omega -$ is a damping coefficient.

Equation (1) arises in various fields of applied physics, nonlinear quantum mechanics, and in the theory of light wave propagation in nonlinear environment (see, for example, [3], [13], [17]). The local well-posedness of problem (1)–(3) was studied in [1]. For $\theta = -1, \omega = 0$ the Cauchy problem in the space R^n for equation (1) with the nonlinear term $f(u)=-i|u|^2 u$ was investigated in [14]. For $\omega=0$ the mixed problem for equation (1) with nonlinear term (3) in the n-dimensional domain $\Omega\subset R^n$ was studied in [7], [9]. The mixed problem for equation (1) for $\omega=0,\ f(u,\nabla u)=\omega_1|u|^{1+\gamma}+\omega_2|u|^{1+\mu}$, where $\omega_1,\omega_2,\gamma,\mu-$ are positive numbers, was studied in the n-dimensional domain $\Omega\subset R^n$ in [10]. The mixed problem for equation (1) for $\omega=0$ in the domain $\Omega\subset R^2$ with the nonlinear term $f(u)=|u|^{\rho}u$, where $\rho>0$, was studied in [4]. The mixed problem in $\Omega\subset R^n$ for equation (1), for $\theta=1,\omega=0$ with the nonlinear term $f(u)=\beta|u|^q u+i\alpha|u|^p u,$ under the condition $q>0, p>0,\alpha\neq 0,\beta\neq 0$, was investigated in [5].

The question of the blow-up of solutions to the Cauchy problem in the space R^n for equation (1) with a nonlinear term $f(u)=-i|u|^{\rho}u$, $\rho>0$, for $\omega=0$ was investigated in works [2], [5], [6], [8], [11], [12], [16]. The question of the nonexistence of global periodic solutions for problem (1)–(3) is also of scientific interest and remains relevant..

The obtained results are formulated in the form of the following theorems.

Theorem 1. Let $\mu>0,\omega>0$. Suppose that $u_0(x)$ satisfies the condition

$$y_0=\frac{1}{L}\int_0^L \operatorname{Im}u_0(x)dx>0.$$

Then the maximal existence time of smooth periodic solutions to problem (1)–(3) is bounded above by the number t_0, $t_{\max}\le t_0$, where

$$t_0=\frac{1}{\omega\rho}\ln\left(\frac{\mu y_0^{\rho}+\omega}{\mu\, y_0^{\rho}}\right).$$

For $\omega<0$ this statement also remains valid provided that

$$t_0=\frac{1}{|\omega|\rho}\ln\left(\frac{\mu y_0^{\rho}}{\mu\, y_0^{\rho}-|\omega|}\right),$$

under the condition

$$y_0>\left(\frac{|\omega|}{\mu}\right)^{1/\rho}.$$

For $\omega = 0$ the quantity t_0 is calculated by the formula

$$t_0 = \frac{1}{y_0^\rho \mu\rho}.$$

Theorem 2. Let $\eta > 0, \omega > 0$. Suppose that $u_0(x)$ satisfies the condition

$$y_0 = \frac{1}{L}\int_0^L \operatorname{Re} u_0(x)dx > 0.$$

Then the maximal existence time of smooth periodic solutions to problem (1)–(3) is bounded above by the number t_0, $t_{\max} \le t_0$, where

$$t_0 = \frac{1}{\omega\rho}\ln\left(\frac{\mu y_0^\rho + \omega}{\mu\, y_0^\rho}\right),$$

For $\omega < 0$ the statement remains valid provided that

$$t_0 = \frac{1}{|\omega|\rho}\ln\left(\frac{\eta y_0^\rho}{\mu\, y_0^\rho - |\omega|}\right),$$

under the condition

$$y_0 > \left(\frac{|\omega|}{\eta}\right)^{1/\rho}.$$

For $\omega = 0$ the quantity t_0 is calculated by the formula

$$t_0 = \frac{1}{y_0^\rho \eta\rho}.$$

We now present an outline of the proof of Theorems 1 and 2. The following lemmas hold.

Lemma 1. Let $\mu > 0, u(x,t)$ be a smooth solution of problem (1)–(3). Suppose tha

$$t\int_0^L \operatorname{Im} u_0(x)dx > 0.$$

Then, for every $t \in [0, t_{\max})$ the function

$$y(t) = \frac{e^{-\omega t}}{L}\int_0^L \operatorname{Im} u(x,t)dx$$

при любом $t \in [0, t_{\max})$ satisfies the following first-order nonlinear differential inequality:

$$\frac{dy}{dt} \geq \mu e^{\omega\rho t} y^{\rho+1}(t), \; y_0 = \frac{1}{L}\int_0^L \operatorname{Im} u_0(x) dx .$$

Proof. We have

$$\frac{dy}{dt} = \frac{1}{L}\operatorname{Im}\int_0^L \left(-\omega e^{-\omega t} u + e^{-\omega t}\frac{\partial u}{\partial t}\right) dx = \frac{e^{-\omega t}}{L}\operatorname{Im}\int_0^L \left(-\omega u + \frac{\partial u}{\partial t}\right) dx =$$

$$= \frac{e^{-\omega t}}{L}\operatorname{Im}\int_0^L \left(i\theta u_{xx} + f(u)\right) dx = \frac{e^{-\omega t}}{L}\operatorname{Im}\int_0^L (\eta + i\mu)|u|^{\rho+1} dx = \mu \frac{e^{-\omega t}}{L}\int_0^L |u|^{\rho+1} dx$$

.

Thus, for the function

$$y(t) = \frac{e^{-\omega t}}{L}\operatorname{Im}\int_0^L u dx \tag{4}$$

the following relation holds for every $t \in [0, t_{\max})$:

$$\frac{dy}{dt} = \mu \frac{e^{-\omega t}}{L}\int_0^L |u|^{\rho+1} dx . \tag{5}$$

Let $\mu > 0$, then

$$\frac{dy}{dt} > 0, \; y(t) \geq y_0, \;\; y_0 = \frac{1}{L}\operatorname{Im}\int_0^L u dx$$

Let $y_0 > 0$. Then $y(t) > 0$. Further, for function (4), by virtue of the estimate $\operatorname{Im} u \leq |u|$ we obtain

$$y(t) \leq \frac{e^{-\omega t}}{L}\int_0^L |u| dx ,$$

hence, by Hölder's inequality for integrals, we conclude that

$$y \leq \frac{e^{-\omega t}}{L}\int_0^L |u| dx \leq \frac{e^{-\omega t}}{L}\left(\int_0^L |u|^{\rho+1} dx\right)^{1/(\rho+1)}\left(\int_0^L dx\right)^{\rho/(\rho+1)} = \frac{e^{-\omega t}}{L}\left(\int_0^L |u|^{\rho+1} dx\right)^{1/(\rho+1)} L^{\rho/(\rho+1)}.$$

Consequently,

$$y^{\rho+1} \leq \frac{e^{-\omega\rho t}}{L} e^{-\omega t}\int_0^L |u|^{\rho+1} dx . \tag{6}$$

It follows from (5) and (6) that, for every $t \in [0, t_{\max})$ the following inequality holds:

$$\frac{dy}{dt} \geq \mu e^{\omega\rho t} y^{\rho+1} ,$$

$$y_0 = \frac{1}{L}\int_0^L \operatorname{Im} u\, dx \quad y_0 > 0.$$

Lemma 2. Let $\eta > 0, u(x,t)$ be a smooth solution to problem (1)-(3). Let

$$\int_0^L \operatorname{Re} u_0(x)dx > 0.$$

Then the function

$$y(t) = \frac{e^{-\omega t}}{L}\int_0^L \operatorname{Re} u(x,t)dx$$

satisfies the following first-order nonlinear differential inequality for every $t \in [0, t_{\max})$:

$$\frac{dy}{dt} \geq \eta e^{\omega \rho t} y^{\rho+1}(t),\ y_0 = \frac{1}{L}\int_0^L \operatorname{Re} u_0(x)dx.$$

The proof of Lemma 2 is analogous to the proof of Lemma 1.

Proof of Theorem 1. From the inequality $dy/dt \geq \mu e^{\omega \rho t} y^{\rho+1}$ for $\omega \neq 0$, separating the variables and integrating, we obtain

$$\int_{y_0}^{y} y^{-\rho-1} dy \geq \mu \int_0^t e^{\omega \rho t} dt,$$

whence the following inequality follows:

$$y^{\rho}(t) \geq \frac{y_0^{\rho}}{1 - (\mu/\omega) y_0^{\rho}\left(e^{\omega \rho t} - 1\right)},$$

from which the statements of Theorem 1 follow.
The case $\omega = 0$ is treated analogously.
The proof of Theorem 2 is analogous to the proof of Theorem 1, taking Lemma 2 into account.
The present work was announced in the note [8].

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